Magic: The Gathering Is Turing Complete

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Abstract

Magic: The Gathering is a popular and famously complicated trading card game about magical combat. In this paper we show that optimal play in real-world *Magic* is at least as hard as the Halting Problem. This provides a positive answer to the question "is there a real-world game where perfect play is undecidable under the rules in which it is typically played?", a question that has been open for a decade [1, 9]. To do this, we present a methodology for embedding an arbitrary Turing machine into a game of *Magic* such that the first player is guaranteed to win the game if and only if the Turing machine halts. Our result applies to how real *Magic* is played, can be achieved using standard-size tournament-legal decks, and does not rely on stochasticity or hidden information. Our result is also highly unusual in that all moves of both players are forced in the construction. This shows that even recognising who will win a game in which neither player has a non-trivial decision to make for the rest of the game is undecidable. We conclude with a discussion of the implications for a unified computational theory of games and remarks about the playability of such a board in a tournament setting.

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1 Introduction

Magic: The Gathering (also known as *Magic*) is a popular trading card game owned by Wizards of the Coast. Formally, it is a two-player zero-sum stochastic card game with imperfect information, putting it in the same category as games like poker and hearts. Unlike those games, players design their own custom decks out of a card-pool of over 20,000 unique cards. *Magic*'s multifaceted strategy has made it a popular topic in artificial intelligence research.

In this paper, we examine *Magic: The Gathering* from the point of view of algorithmic game theory, looking at the computational complexity of evaluating who will win a game. As most games have finite limits on their complexity (such as the size of a game board)

most research in algorithmic game theory of real-world games has primarily looked at generalisations of commonly played games rather than the real-world versions of the games. A few real-world games have been found to have non-trivial complexity, including *Dots-and-Boxes*, *Jenga* and *Tetris* [7]. We believe that no real-world game is known to be harder than NP previous to this work.

Even when looking at generalised games, very few examples of undecidable games are known. On an abstract level, the Team Computation Game [8] shows that some games can be undecidable, if they are a particular kind of team game with imperfect information. The authors also present an equivalent construction in their Constraint Logic framework that was used by Coulombe and Lynch (2018) [6] to show that some video games, including *Super Smash Bros Melee* and *Mario Kart*, have undecidable generalisations. Constraint Logic is a highly successful and highly flexible framework for modelling games as computations.

The purpose of this paper is to present the first proof that a game, *as it is actually played in the world*, can be undecidable. The core of this paper is the construction presented in Section 4: a universal Turing machine embedded into a game of *Magic: The Gathering*. As we can arrange for the victor of the game to be determined by the halting behaviour of the Turing machine, this construction establishes the following theorem:

▶ **Theorem 1.** *Determining the outcome of a game of Magic: The Gathering in which all remaining moves are forced is undecidable.*

1.1 Previous Work

Prior to this work, no undecidable real games were known to exist. Demaine and Hearn (2009) [9] note that almost every real-world game is trivially decidable, as they produce game trees with only computable paths. They further note that Rengo Kriegspiel[1] is "a game humans play that is not obviously decidable; we are not aware of any other such game." It is conjectured by Auger and Teytaud (2012) [1] that Rengo Kriegspiel is in fact undecidable, and it is posed as an open problem to demonstrate any real game that is undecidable.

The approach of embedding a Turing machine inside a game directly is generally not considered to be feasible for real-world games [9]. Although some open-world sandbox digital games such as Minecraft and Dwarf Fortress can support the construction of Turing machines, those machines have no strategic relevance and those games are deliberately designed to support large-scale simulation. In contrast, leading formal theory of strategic games claims that the unbounded memory required to simulate a Turing machine entirely in a game would be a violation of the very nature of a game [8].

The computational complexity of *Magic: The Gathering* has been studied previously by several authors. Our work is inspired by [4], in which it was shown that four-player *Magic* can simulate a Turing machine under certain assumptions about player behaviour. In that work, Churchill conjectures that these limitations can be removed and preliminary work along those lines is discussed in [5]. The computational complexity of checking the legality of a particular decision in *Magic* (blocking) is investigated in [3] and is found to be coNP-complete. There have also been a number of papers investigating algorithmic and artificial intelligence approaches to playing *Magic*, including Ward and Cowling (2009) [14], Cowling et al. (2012) [15], and Esche (2018) [10]. Esche (2018) briefly considers the theoretical computational complexity of *Magic* and states an open problem that has a positive answer only if *Magic* end-games are decidable.

[1] Rengo Kriegspiel is a combination of two variations on Go: Rengo, in which two players play on a team alternating turns, and Shadow Go, in which players are only able to see their own moves.

1.2 Our Contribution

This paper completes the project started by Churchill (2012) [4] and continued by Churchill et al. (2014) [5] of embedding a universal Turing machine in *Magic: The Gathering* such that determining the outcome of the game is equivalent to determining the halting of the Turing machine. This is the first result showing that there exists a real-world game for which determining the winning strategy is non-computable, answering an open question of Demaine and Hearn [9] and Auger and Teytaud [1] in the positive.

Our result is also a contribution to the existing literature on the computational complexity of *Magic: the Gathering* [4, 3, 10]. Combined with Rice's Theorem [12], it also answers an open problem from Esche [10] in the negative by showing that the equivalence of two strategies for playing *Magic* is undecidable.

This result raises important foundational questions about the nature of a game itself. As we have already discussed, the leading formal theory of games holds that this construction is unreasonable, if not impossible, and so a reconsideration of those assumptions is called for. In section 5.1 we discuss additional foundational assumptions of Constraint Logic that *Magic: The Gathering* violates, and present our interpretation of the implications for a unified theory of games.

1.3 Overview

The paper is structured as follows. In Section 2 we provide background information on this work, including previous work on *Magic* Turing machines. In Section 3 we present a sketch of the construction and its key pieces. In Section 4 we provide the full construction of a universal Turing machine embedded in a two-player game of *Magic*. In Section 5 we discuss the game-theoretic and real-world implications of our result.

Appendix A provides a brief overview of the relevant rules to *Magic* for those who are not familiar with the game.

2 Preliminaries

One initial challenge with *Magic: The Gathering* is the encoding of information. Some cards ask players to choose a number. Although rules for how to specify a number are not discussed in the *Comprehensive Rules* [16], convention is that players are allowed to specify numbers in any way that both players can agree to. For example, you are allowed to choose the number 2^{100} or $\lceil \log 177 \rceil$. This presents an issue brought to our attention by Fortanely [11]. Consider the following situation: both players control **Lich**, **Transcendence**, and **Laboratory Maniac**. One player then casts **Menacing Ogre**. The net effect of this play is the "Who Can Name the Bigger Number" game – whoever picks the biggest number wins on the spot. If players are allowed to use sufficiently complicated (but well-defined) functions to express their choices, identifying the next board state can be non-computable even given the players' choices [2]. To remedy this situation, we require that any numbers specified by a player must be expressed in standard binary notation[2].

We believe that with this restriction *Magic: The Gathering* is *transition-computable*, meaning that the function that maps a board state and a move to the next board state is computable[3]. However, it is unclear how to prove this beyond exhaustive analysis of the over 20,000 unique cards in the game. We leave that question open for future work:

[2] Only integers exist in *Magic*. If a result would be a non-integer, it is rounded.
[3] We avoid the term "computable game" which is more commonly used to mean that the game has a computable winning strategy.

▶ Conjecture 1. The function that takes a board state and a legal move and returns the next board state in *Magic: The Gathering* is computable.

In this conjecture we say "a legal move" because it is also not obvious that checking to see if a move is legal is computable. Chatterjee and Ibsen-Jensen [3] show that checking the legality of a particular kind of game action in *Magic: the Gathering* is coNP-complete, but the question has not been otherwise considered. Again, we leave this for future work:

▶ Conjecture 2. There does not exist an algorithm for checking the legality of a move in *Magic: The Gathering*.

2.1 Previous *Magic* Turing Machines

In Churchill (2012) [4], the author presents a *Magic: The Gathering* end-game that embeds a universal Turing machine. However, this work has a major issue from a computability theory point of view: it's not quite deterministic. At several points in the simulation, players have the ability to stop the computation at any time by opting to decline to use effects that say "may." For example, **Kazuul Warlord** reads "Whenever Kazuul Warlord or another Ally enters the battlefield under your control, you may put a $+1/+1$ counter on each Ally you control." Declining to use this ability will interfere with the Turing machine, either causing it to stop or causing it to perform a different calculation from the one intended. The construction as given in Churchill (2012)[4] works under the assumption that all players that are given the option to do something actually do it, but as the author notes it fails without this assumption. Attempts to correct this issue are discussed in Churchill et al. [5].

In this work, we solve this problem by reformulating the construction to exclusively use cards with mandatory effects. We also substantially simplify the most complicated aspect of the construction, the recording of the tape, and reduce the construction from one involving four players to one involving two, and which only places constraints on one player's deck, matching the format in which *Magic* is most commonly played in the real world (two-player duels). Like the previous work, we will embed Rogozhin's (2, 18) universal Turing machine [13].

3 An Overview of the Construction

In this section we give a big picture view of the Turing machine, with full details deferred to the next section[4]. The two players in the game are named Alice and Bob.

To construct a Turing machine in *Magic: The Gathering* requires three main elements: the tape which encodes the computation, the controller which determines what action to take next based on the current state and the last read cell, and the read/write head which interacts with the tape under the control of the controller.

3.1 The Tape

As the rules of *Magic: The Gathering* do not contain any concept of geometry or adjacency, encoding the tape itself is tricky. Our solution is to have many creature tokens with carefully controlled power and toughness, with each token's power and toughness representing the distance from the head of the Turing machine. The tape to the left of the Turing machine's current read head position is represented by a series of creature tokens which all have

[4] A walkthrough of the construction

the game colour green, while the tape to the right is represented by white tokens. Our distance-counting starts at 2, so there is one 2/2 token representing the space currently under the head of the Turing machine; a green 3/3 token represents the tape space immediately to the left of the Turing head, a green 4/4 is the space to the left of that, and so on. Rogozhin's universal Turing machine starts with the read head in the middle of the tape [13].

To represent the symbols on the tape, we use creature types. We choose 18 creature types from the list of creature types in *Magic* to correspond to the 18 symbols in Rogozhin's (2, 18) UTM. We can choose these creature types to begin with successive letters of the alphabet: Aetherborn, Basilisk, Cephalid, Demon, Elf, Faerie, Giant, Harpy, Illusion, Juggernaut, Kavu, Leviathan, Myr, Noggle, Orc, Pegasus, Rhino, and Sliver. For example, a green 5/5 Aetherborn token represents that the 1st symbol is written on the 3rd cell to the left of the head, and a white 10/10 Sliver represents that the 18th symbol is written on the 8th cell to the right of the head. These tokens are all controlled by Bob, except the most recently created token (the space the Turing head has just left) which is controlled by Alice.

3.2 The Controller

Control instructions in a Turing machine are represented by a table of conditional statements of the form "if the machine is in state s, and the last read cell is symbol k, then do such-and-such." Many *Magic* cards have triggered abilities which can function like conditional statements. The two we shall use are **Rotlung Reanimator** ("Whenever Rotlung Reanimator or another Cleric dies, create a 2/2 black Zombie creature token") and **Xathrid Necromancer** ("Whenever Xathrid Necromancer or another Human creature you control dies, create a tapped 2/2 black Zombie creature token"). We will use both, and the difference between tapped and untapped creature tokens will contribute to the design of the Turing machine[5].

Each **Rotlung Reanimator**[6] needs to trigger from a different state being read – that is, a different creature type dying – and needs to encode a different result. Fortunately, *Magic* includes cards that can be used to edit the text of other cards. The card **Artificial Evolution** is uniquely powerful for our purposes, as it reads "Change the text of target spell or permanent by replacing all instances of one creature type with another. The new creature type can't be Wall. *(This effect lasts indefinitely.)*" So we create a large number of copies of **Rotlung Reanimator** and edit each one. A similar card, **Glamerdye**, can be used to modify the colour words within card text.

Thus, we edit a **Rotlung Reanimator** by casting two copies of **Artificial Evolution** replacing "Cleric" with "Aetherborn" and "Zombie" with "Sliver" and one copy of **Glamerdye** to replace "black" with "white", so that this **Rotlung Reanimator** now reads "Whenever Rotlung Reanimator or another *Aetherborn* dies, create a 2/2 *white Sliver* creature token"[7]. This **Rotlung Reanimator** now encodes the first rule of the q_1 program of the (2, 18) UTM: "When reading symbol 1 in state A, write symbol 18 and move left." The Aetherborn creature token represents symbol 1, the Sliver creature token represents symbol 18, and the fact that the token is white leads to processing that will cause the head to move left.

We similarly have seventeen more **Rotlung Reanimator**s encoding the rest of the q_1 program from [13]. Between them they say:

[5] See Appendix A.3 for information about tapping

[6] For now we will speak about **Rotlung Reanimator** for simplicity. Some of these will in fact be **Xathrid Necromancer**s as explained in the next section.

[7] Throughout this paper, card text that has been modified is written in italics.

1. Whenever an *Aetherborn* dies, create a 2/2 *white Sliver*.
2. Whenever a *Basilisk* dies, create a 2/2 *green Elf*.
Whenever a ... dies, create a 2/2 ...
18. Whenever a *Sliver* dies, create a 2/2 *green Cephalid*.
See Table 2 in Appendix B for the full encoding of the program.

3.3 The Read/Write Head

The operation "read the current cell of the tape" is represented in-game by forcing Alice to cast **Infest** ("All creatures get –2/–2 until end of turn."), which causes the unique token with 2 toughness to die. It had a colour (green or white) which is irrelevant, and a creature type which corresponds to the symbol written on that cell. That creature type is noticed by a **Rotlung Reanimator**, which has a triggered ability that is used to carry out the logic encoded in the head of the Turing machine. It produces a new 2/2 token, containing the information written to the cell that was just read.

The Turing machine then moves either left or right and modifies the tokens to keep the tape in order by adding $+1/+1$ counters to all tokens on one side of the head and –1/–1 counters to all tokens on the other side. Moving left or right will be accomplished by casting first **Cleansing Beam** ("Radiance – Cleansing Beam deals 2 damage to target creature and each other creature that shares a colour with it.") and then **Soul Snuffers** ("When Soul Snuffers enters the battlefield, put a -1/-1 counter on each creature").

3.4 Adding a Second State

Everything described so far outlines the operation of one state of the Turing machine. However, our Turing machine requires two states. To accomplish this, we leverage *phasing*. An object with phasing can "phase in" or "phase out", and while it's phased out, it's treated as though it doesn't exist. We can grant phasing to our **Rotlung Reanimator**s using the enchantment **Cloak of Invisibility** ("Enchanted creature has phasing and can't be blocked except by Walls") and create a second set of **Rotlung Reanimator**s to encode the program q_2. This second set will also be granted phasing in this way, and the two programs will be set to be in opposition, with one phased out and one phased in at every point in time. At the moment we read the current cell, the set of **Rotlung Reanimator**s that is phased in is determined by which state we are in.

Objects with phasing phase in or out at the beginning of their controller's turn, effectively toggling between two states. Accordingly we will arrange for the turn cycle to last 4 turns for each player when no state change occurs, but just 3 turns when we need to change state.

4 The Full Construction

Now we will provide the full construction of the *Magic: The Gathering* Turing machine and walk through a computational step. The outline of one step of the computation is as follows (Bob's turns are omitted as nothing happens during them):

T1 Alice casts **Infest**. Turing processing occurs: a white or green token dies, a new white or green token is created.
T2 Alice casts **Cleansing Beam**, putting two $+1/+1$ counters on the side of the tape we are moving away from.
T3 If the Turing machine is remaining in the same state, Alice casts **Coalition Victory**. If it is changing state, Alice casts **Soul Snuffers**, putting a –1/–1 counter on each creature.
T4 If the Turing machine is remaining in the same state, this is the point where Alice casts **Soul Snuffers**. Otherwise, the next computational step begins.

After all four turns have passed, the Turing machine has finished processing for one step of the computation and moves on to the next.

4.1 Beginning a Computational Step and Casting Spells

At the beginning of a computational step, it is Alice's turn and she has the card **Infest** in hand. Her library consists of the other cards she will cast during the computation (**Cleansing Beam**, **Coalition Victory**, and **Soul Snuffers**, in that order). Bob's hand and library are both empty. The Turing machine is in its starting state and the tape has already been initialised.

At the start of each of Alice's turns, she has one card in hand. She's forced to cast it due to Bob controlling **Wild Evocation**, which reads "At the beginning of each player's upkeep, that player reveals a card at random from their hand. If it's a land card, the player puts it onto the battlefield. Otherwise, the player casts it without paying its mana cost if able." When the card resolves, it would normally be put into her graveyard, but Alice is enchanted by **Wheel of Sun and Moon** ("Enchant player. If a card would be put into enchanted player's graveyard from anywhere, instead that card is revealed and put on the bottom of that player's library."), which causes it to be placed at the bottom of her library instead, allowing her to redraw it in the future and keeping the cards she needs to cast in order. After her upkeep step, Alice proceeds to her draw step and draws the card that she will cast next turn.

Alice has no choices throughout this process: she does control one land, but it remains permanently tapped because of **Choke** ("Islands don't untap during their controllers' untap steps"), so she is unable to cast any of the spells she draws except via **Wild Evocation**'s ability. Neither player is able to attack because they both control a **Blazing Archon** ("Creatures can't attack you.").

Bob has no cards in hand and controls **Recycle**, which reads (in part) "Skip your draw step". This prevents Bob from losing due to drawing from an empty library.

4.2 Reading the Current Cell

On the first turn of the cycle, Alice is forced to cast **Infest** ("All creatures get –2/–2 until end of turn."). This kills one creature: the tape token at the position of the current read head, controlled by Bob. This will cause precisely one creature of Bob's to trigger – either a **Rotlung Reanimator** or a **Xathrid Necromancer**. Which precise one triggers is based on that token's creature type and the machine's current state, corresponding to the appropriate rule in the definition of the (2, 18) UTM. This Reanimator or Necromancer will create a new 2/2 token to replace the one that died. The new token's creature type represents the symbol to be written to the current cell, and the new token's colour indicates the direction for the machine to move: white for left or green for right.

Alice controls **Illusory Gains**, an Aura which reads "You control enchanted creature. Whenever a creature enters the battlefield under an opponent's control, attach Illusory Gains to that creature." Each time one of Bob's **Rotlung Reanimator**s or **Xathrid Necromancer**s creates a new token, **Illusory Gains** triggers, granting Alice control of the newest token on the tape, and reverting control of the previous token to Bob. So at any point Bob controls all of the tape except for the most recently written symbol, which is controlled by Alice.

4.3 Moving Left or Right

If the new token is white, the Turing machine needs to move left. To do this we need to take two actions: put a $+1/+1$ counter on all white creatures (move the tape away from white), and put a –1/–1 counter on all green creatures (move the tape towards green). We rephrase this instead as: put two $+1/+1$ counters on all white creatures, and put a –1/–1 counter on *all* creatures.

On Alice's second turn, she casts **Cleansing Beam**, which reads "Cleansing Beam deals 2 damage to target creature and each other creature that shares a colour with it." Bob controls **Privileged Position** ("Other permanents you control have hexproof. (They can't be the targets of spells or abilities your opponents control.)") so none of Bob's creatures are a legal target. Alice controls some creatures other than the tape token, but they have all been granted creature type Assembly-Worker by a hacked **Olivia Voldaren**, and Alice controls a **Steely Resolve** naming Assembly-Worker ("Creatures of the chosen type have shroud. *(They can't be the targets of spells or abilities.)*") This makes it so that the only legal target for **Cleansing Beam** is the one tape token that Alice controls thanks to her **Illusory Gains**.

Recall that this token is white if we're moving left, or green if we're moving right. **Cleansing Beam** is about to deal 2 damage to each white creature if we're moving left, or to each green creature if we're moving right. Alice and Bob each control a copy of **Vigor** ("If damage would be dealt to another creature you control, prevent that damage. Put a $+1/+1$ counter on that creature for each 1 damage prevented this way."), so **Cleansing Beam** ends up putting two $+1/+1$ counters on either each white creature or each green creature.

On the last turn of the cycle, Alice casts **Soul Snuffers**, a 3/3 black creature which reads "When Soul Snuffers enters the battlefield, put a –1/–1 counter on each creature." There are two copies of **Dread of Night** hacked to each say "*Black* creatures get –1/–1", which mean that the **Soul Snuffers**' triggered ability will kill itself, as well as shrinking every other creature. The creatures comprising the tape have now received either a single –1/–1 counter, or two $+1/+1$ counters and a –1/–1 counter.

To ensure that the creatures providing the infrastructure (such as **Rotlung Reanimator**) aren't killed by the succession of –1/–1 counters each computational step, we arrange that they also have game colours green, white, red and black, using **Prismatic Lace**, "Target permanent becomes the colour or colours of your choice. *(This effect lasts indefinitely.)*" Accordingly, each cycle **Cleansing Beam** will put two $+1/+1$ counters on them, growing them faster than the –1/–1 counters shrink them. This applies to each creature except **Vigor** itself; to keep each player's **Vigor** from dwindling, there is a **Fungus Sliver** hacked to read "All *Incarnation* creatures have 'Whenever this creature is dealt damage, put a $+1/+1$ counter on it.' "

4.4 Changing State

The instruction to change state is handled by replacing seven of Bob's **Rotlung Reanimator**s with **Xathrid Necromancer**. These two cards have very similar text, except that **Xathrid Necromancer** only notices Bob's creatures dying (this is not a problem, as the active cell of the tape is always controlled by Bob), and that **Xathrid Necromancer** creates its token *tapped*.

For example, when the q_1 program (State A) sees symbol 1, it writes symbol 18, moves left, and remains in state A. This is represented by a phasing **Rotlung Reanimator** under Bob's control saying "Whenever Rotlung Reanimator or another *Aetherborn* dies, create a 2/2 *white Sliver* creature token."

By contrast, when the q_1 program sees symbol 11, it writes symbol 12, moves left, and changes to state B. This is represented by a phasing **Xathrid Necromancer** under Bob's control saying "Whenever Xathrid Necromancer or another *Kavu* creature you control dies, create a tapped 2/2 *white Leviathan* creature token."

In both cases this token is created under Bob's control on turn T1, but Alice's **Illusory Gains** triggers and grants her control of it. In the case where it's tapped, that means at the beginning of turn T2, it will untap. This causes Alice's **Mesmeric Orb** ("Whenever a permanent becomes untapped, that permanent's controller puts the top card of his or her library into his or her graveyard.") to trigger, and that ability to be put on the stack at the same time as Bob's **Wild Evocation**'s trigger (since no player receives priority during the untap step). Alice is the active player, so Alice's trigger is put on the stack first and then Bob's[16]. This ensures that the **Wild Evocation** trigger resolves, forcing Alice to cast and resolve **Cleansing Beam**, before the **Mesmeric Orb** trigger resolves.

When the **Mesmeric Orb** trigger does resolve, it tries to put the **Coalition Victory** from the top of Alice's library into her graveyard. **Wheel of Sun and Moon** modifies this event to put **Coalition Victory** onto the bottom of her library, just underneath the **Cleansing Beam** that's just resolved.

Once all these triggers are resolved, Alice proceeds to her draw step. When the state is not changing, she will draw **Coalition Victory** at this point, but when the state is changing, that card is skipped and she moves on to draw **Soul Snuffers**.

The net result of this is that the computation step is 3 turns long for each player when the state is changing, but 4 turns long for each player when the state is not changing. In the normal 4-turn operation, Bob's phasing Reanimators and Necromancers will phase in twice and phase out twice, and be in the same state on one cycle's turn T1 as they were in the previous cycle's turn T1. But when changing state, they will have changed phase by the next cycle's turn T1, switching the Turing machine's state.

4.5 Out of Tape

The Turing tape can be initialised to any desired length before starting processing. But it is preferable to allow the machine to run on a simulated infinite tape: in other words, to assume that any uninitialised tape space contains symbol 3 (the blank symbol in the (2, 18) UTM), represented by creature type Cephalid. This is accomplished by having the ends of the currently-initialised tape marked by two special tokens, one green Lhurgoyf and one white Rat.

Suppose we've exhausted all the initialised tape to the left. This means that the casting of **Infest** on turn T1 kills the Lhurgoyf rather than one of the normal tape types. This does not directly trigger any of the normal Reanimators/Necromancers. Instead, Bob has another **Rotlung Reanimator** hacked to read "Whenever Rotlung Reanimator or another *Lhurgoyf* dies, create a 2/2 *green Lhurgoyf* creature token", and Alice has a **Rotlung Reanimator** hacked to read "Whenever Rotlung Reanimator or another *Lhurgoyf* dies, create a 2/2 *black Cephalid* creature token." Bob's trigger will resolve first, then Alice's.

First, Bob's Reanimator trigger creates a new Lhurgoyf just to the left of the current head. (Alice's **Illusory Gains** triggers and gives her control of this new Lhurgoyf, but that will soon change.) We have one copy of **Shared Triumph** set to Lhurgoyf ("Creatures of the chosen type get +1/+1") so this token arrives as a 3/3.

Second, Alice's Reanimator trigger now creates a 2/2 black Cephalid under Alice's control. The same two copies of **Dread of Night** as before are giving all black creatures –2/–2, so the black Cephalid will arrive as a 0/0 and immediately die.

The death of this Cephalid triggers one of the regular **Rotlung Reanimator**s of Bob's just as if a tape cell containing symbol 3 had been read: a new 2/2 token is created and **Illusory Gains** moves to that new token. The green Lhurgoyf token serving as an end-of-tape marker has been recreated one step over to the left.

The situation for the white Rat representing the right-hand end of the tape is exactly equivalent. Bob has a **Rotlung Reanimator** hacked to read "Whenever Rotlung Reanimator or another *Rat* dies, create a 2/2 *white Rat* creature token"; Alice has a **Rotlung Reanimator** hacked to read "Whenever Rotlung Reanimator or another *Rat* dies, create a 2/2 *black Cephalid* creature token"; and we have another **Shared Triumph** set to Rat. This algorithm would be a little more complex if reading symbol 3 could cause a state change in the (2, 18) UTM, but thankfully it cannot.

4.6 Halting

We choose to encode halting as making Alice win the game. When the Turing machine doesn't change state, Alice casts the card **Coalition Victory** on her third turn. It reads "You win the game if you control a land of each basic land type and a creature of each colour." This normally accomplishes nothing because she controls no blue creatures (**Prismatic Lace** has been used to give her creatures of all the other colours). She does, however, control one land, and also controls **Prismatic Omen**, which reads "Lands you control are every basic land type in addition to their other types."

When the halt symbol is read (symbol 17 in state A), the appropriate phasing Reanimator of Bob's reads "Whenever Rotlung Reanimator or another *Rhino* dies, create a 2/2 *blue Assassin* creature token." Alice's **Illusory Gains** takes control of this Assassin token in the usual way in turn T1. She now meets the condition for **Coalition Victory** when she casts it on turn T3, and wins the game. As the token is created by a **Rotlung Reanimator**, it will be untapped and so Alice will not be made to discard **Coalition Victory**.

If the encoded machine does not in fact halt then the game has entered an unbreakable deterministic infinite loop, which is specified as a draw by rule 104.4b [16].

5 Discussion

The construction in the previous section establishes Theorem 1: determining the outcome of a game of *Magic: The Gathering* in which all remaining moves are forced is undecidable. As determining the existence of a winning move is no harder than optimal play, this also establishes that optimal play in *Magic* is at least as hard as the halting problem.

The full complexity of optimal strategic play remains an open question, as do many other computational aspects of *Magic*. For example, a player appears to have infinitely many moves available to them from some board states of *Magic*. Whether or not there exists a real-world game of *Magic* in which a player has infinitely many *meaningfully different* moves available to them has the potential to impact the way we understand and model games as a form of computation. As far as we are aware, no existing computational models of games support games with infinitely many possible moves from a given board state.

5.1 Consequences for Computational Theories of Games

This construction establishes that *Magic: The Gathering* is the most computationally complex real-world game known in the literature, but it also raises several foundational questions about games as a form of computation. Some authors, such as Demaine and Hearn [8],

have sought a formal framework for modelling games that is strictly sub-Turing. Unlike the open-world, non-strategic games in which Turing machines have been constructed before, *Magic: The Gathering* is unambiguously a two-player strategic game like such models attempt to represent. Therefore this result shows that any sub-Turing model is necessarily inadequate to capture all strategic games. Quite the opposite: it seems likely that a *super-Turing* model of games would be necessary to explain *Magic*. The naïve extensions of Demaine and Hearn's Constraint Logic to allow for unbounded memory appear to be meaningless, although it's possible that a clever approach would bring success.

▶ Open Problem 3. Does there exist a generalisation of Constraint Logic that explains the computational complexity of *Magic: The Gathering*?

Although the Halting problem is reducible to our construction, the fact that even evaluating a board is non-computable strongly suggests that the complexity of strategic play is greater than that. In particular, we conjecture:

▶ Conjecture 4. Playing *Magic: The Gathering* optimally is at least as hard as $\emptyset^{(\omega)}$.

Whether or not it is possible for there to be a real-world game whose computational complexity is strictly harder than $\emptyset^{(\omega)}$ is an interesting question on both a mathematical and a philosophical level. If not, then this conjecture would imply that *Magic: The Gathering* is as hard as it is possible for a real-world game to be.

5.2 Real-world Playability and Legality

While there are practical difficulties involved with correctly setting up the necessary board state, such as running out of space on your table, a sufficiently tenacious player could set up and execute this construction in a real-world tournament game of *Magic: The Gathering*. An example 60-card deck that is capable of executing this construction on the first turn of the game and which is legal in the competitive Legacy format can be seen in Table 3 in Appendix B.

With the correct draw, the deck uses **Ancient Tomb** ("tap: Add two colorless mana to your mana pool. Ancient Tomb deals 2 damage to you.") and three **Lotus Petal**s ("tap, Sacrifice Lotus Petal: Add one mana of any color to your mana pool. Play this ability as a mana source.") to play **Grim Monolith** ("Grim Monolith does not untap during your untap phase. T: Add three colorless mana to your mana pool. Play this ability as a mana source.") and **Power Artifact** ("Enchantment - Aura. Enchant artifact Enchanted artifact's activated abilities cost less to activate. This effect can't reduce the amount of mana an ability costs to activate to less than one mana.") and generate unlimited colourless mana, at which point **Staff of Domination** ("5, tap: Draw a card.") draws the rest of the deck and **Gemstone Array** ("2: Put a charge counter on Gemstone Array. Remove a charge counter from Gemstone Array: Add one mana of any color.") generates unlimited coloured mana. The deck casts most of the permanents immediately, and uses **Stolen Identity** ("Create a token that's a copy of target artifact or creature.") to make token copies of them (using **Memnarch** ("1UU: Target permanent becomes an artifact in addition to its other types.") first on the enchantments like **Cloak of Invisibility**). The tape is initialised with **Riptide Replicator** ("As Riptide Replicator enters the battlefield, choose a color and a creature type. Riptide Replicator enters the battlefield with X charge counters on it. 4, tap: Create an X/X creature token of the chosen color and type, where X is the number of charge counters on Riptide Replicator.") and **Capsize** ("Buyback 3 Return target permanent to its owner's hand."), **Reito Lanturn** ("3: Put target card in a graveyard on the bottom of its owner's library") allows repeated casting of the text-modification cards,

Reality Ripple ("Target artifact, creature, or land phases out.") sets the phase of the **Rotlung Reanimator**s and **Donate** ("Target player gains control of target permanent you control.") gives most permanents to Bob. Once the board is set up, **Reito Lantern** is also used to get Alice's library into the correct order. Finally, **Steely Resolve** ("As Steely Resolve enters the battlefield, choose a creature type. Creatures of the chosen type have shroud.") is cast, and then **Karn Liberated** and **Capsize** are used to exile all setup permanents and all cards from both players' hands so that neither player can interact with the operation of the Turing machine.

In addition to the *Comprehensive Rules* [16], play at sanctioned *Magic: The Gathering* tournaments is also governed by the *Tournament Rules* [17]. Some of these rules, most notably the ones involving slow play, may affect an individual's ability to successfully execute the combo due to concerns about the sheer amount of time it would take to manually move the tokens around to simulate a computation on a Turing machine. This would not be a concern for two agents with sufficiently high computational power, as the Tournament Rules also provide a mechanism called "shortcuts" for players to skip carrying out laborious loops if both players agree on the game state at the beginning and the end of the shortcut.

6 Conclusion

We have presented a methodology for embedding Rogozhin's (2, 18) universal Turing machine in a two-player game of *Magic: The Gathering*. Consequently, we have shown that identifying the outcome of a game of *Magic* in which all moves are forced for the rest of the game is undecidable. In addition to solving a decade-old outstanding open problem, in the process of arriving at our result we showed that *Magic: The Gathering* does not fit assumptions commonly made by computer scientists while modelling games. We conjecture that optimal play in *Magic* is far harder than this result alone implies, and leave the true complexity of *Magic* and the reconciliation of *Magic* with existing theories of games for future research.

References

1 David Auger and Oliver Teytaud. The frontier of decidability in partially observable recursive games. *International Journal of Foundations of Computer Science*, 2012.

2 Stella Biderman and Bjørn Kjos-Hanssen. Non-comparable natural numbers. Theoretical Computer Science Stack Exchange, 2018. URL: `https://cstheory.stackexchange.com/q/41384(version:2018-08-16)`.

3 Krishnendu Chatterjee and Rasmus Ibsen-Jensen. The complexity of deciding legality of a single step of Magic: The Gathering. In *22nd European Conference on Artificial Intelligence*, 2016.

4 Alex Churchill. *Magic: The Gathering* is Turing complete v5, 2012. URL: `https://www.toothycat.net/~hologram/Turing/`.

5 Alex Churchill et al. Magic is Turing complete (the Turing machine combo), 2014. URL: `http://tinyurl.com/pv3n2lg`.

6 Michael J. Coulombe and Jayson Lynch. Cooperating in video games? Impossible! Undecidability of team multiplayer games. In *9th International Conference on Fun with Algorithms*, 2018.

7 Erik D. Demaine and Robert A. Hearn. Playing games with algorithms: algorithmic combinatorial game theory. In *26th Symp. on Mathematical Foundations in Computer Science*, pages 18–32, 2001.

8 Erik D. Demaine and Robert A. Hearn. Constraint logic: A uniform framework for modeling computation as games. In *2008 23rd Annual IEEE Conference on Computational Complexity*, pages 149–162, 2008.

9 Erik D. Demaine and Robert A. Hearn. *Games, Puzzles, and Computation*. CRC Press, 2009.

10 Alexander Esche. *Mathematical Programming and Magic: The Gathering*. PhD thesis, Northern Illinois University, 2018.

11 Eugenio Fortanely. Personal communication, 2018.

12 H. G. Rice. Classes of recursively enumerable sets and their decision problems. *Trans. Amer. Math. Soc.*, 74:358–366, 1953.

13 Yurii Rogozhin. Small universal Turing machines. *Theoretical Computer Science*, 168(2):215–240, 1996.

14 Colin D. Ward and Peter I. Cowling. Monte Carlo search applied to card selection in Magic: The Gathering. In *CIG'09 Proceedings of the 5th international conference on Computational Intelligence and Games*, pages 9–16, 2009.

15 Colin D. Ward, Peter I. Cowling, and Edward J. Powley. Ensemble determinization in Monte Carlo tree search for the imperfect information card game Magic: The Gathering. In *IEEE Transactions on Computational Intelligence and AI in Games*, volume 4, 2012.

16 Wizards of the Coast. Magic: The Gathering comprehensive rules, August 2018. URL: `https://magic.wizards.com/en/game-info/gameplay/rules-and-formats/rules`.

17 Wizards of the Coast. Magic: The Gathering tournament rules, August 2018. URL: `https://wpn.wizards.com/sites/wpn/files/attachements/mtg_mtr_21jan19_en.pdf`.

A How to Play *Magic: the Gathering*

In this section we provide a brief overview of the game and its rules, with a focus on what is necessary to understand the Turing machine construction. The full *Magic: the Gathering* Comprehensive Rules document [16] is over 200 pages of text and detailing them falls outside the purview of this paper.

A.1 An Introduction to *Magic*

Magic: the Gathering is a tabletop card game about magical combat. Each player brings their own deck of cards that they've chosen, called their *library*. On a player's turn, that player plays cards to cast spells, summon creatures, and use abilities of cards they've already played. When creatures die or when one-time effects are used, those cards are placed in a discard pile called the *graveyard*. Each player begins with 20 *life points* and once they are depleted that player loses the game; the main way of reducing the opponent's life total is to attack with creatures. A player can also lose the game if they attempt to draw from a library with no cards remaining.

Cards have various *types*. Card types include:

1. **Creature**. Creature cards are *permanents*, which means they are played onto the table (the *battlefield*) and remain in play. Creatures have two important statistics: their *power*, representing how much damage they can deal in combat, and their *toughness*, representing how much health they have or how far they are from dying. Standard notation for these uses a slash: a 3/2 creature has 3 power and 2 toughness. If a creature's toughness is ever zero or less, that creature *dies* and is put into the graveyard.
2. **Artifact** and **Enchantment**. These are also permanents. They do not engage in combat but have abilities that affect players or other permanents. Some enchantments are *Auras*: these are attached to one other permanent or player and have an effect on that permanent or player. There are minor differences between artifacts and enchantments but they are not relevant to our construction.

3. **Instant** and **Sorcery**. These are cards that have a one-time effect and are then immediately put into the graveyard. They may cause effects that last until the end of the turn or which last indefinitely.
4. **Land**: Land cards remain on the battlefield. Usually they provide the resource known as *mana* which is used to play other cards. In our construction we have one land card but we ensure its controller cannot activate its ability.

Some cards have subtypes in addition to types. Aura is an example of a subtype of enchantments. All creatures have subtypes called *creature types* such as Goblin or Wizard that denote their race or class, and those creature types are used in various ways throughout the construction, in particular to track the symbols written onto the Turing tape.

A.2 Tokens

Some effects can create *tokens* on the battlefield, which are also permanents. This is crucial to the construction of a Turing tape potentially millions of cells long with a bounded number of cards. Tokens may be creatures, generally with no abilities, or they may be copies of other permanents such as enchantments or artifacts. Unless an effect specifies otherwise, tokens are treated exactly like cards of the same type while they are on the battlefield.

Tokens can only exist on the battlefield – if they ever leave the battlefield they cease to exist. If a creature token dies, it leaves the battlefield and goes to the graveyard (triggering any effects that watch for those conditions such as **Rotlung Reanimator**'s). However, it does not continue to exist in the graveyard.

A.3 Tapping

"Tapping" is a core mechanic in *Magic: the gathering* typically represented by turning a card 90 degrees. Being tapped is a binary state: a permanent is either tapped or untapped. At the beginning of each player's turn, the very first thing they do is untap all tapped permanents that they control. While there are a variety of ways that permanents can become tapped (some of which are used to set up the device), in the operation of the *Magic* Turing machine permanents will never become tapped. Tokens will be created either in a tapped or in an untapped state, and the difference between tapped and untapped tokens will control the state of the *Magic* Turing machine by controlling phasing.

A.4 Editing Card Text and Types

The Turing machine construction is only possible because certain *Magic* cards allow modification of the text of other cards, to change colours or creature types. The card **Artificial Evolution** reads "Change the text of target spell or permanent by replacing all instances of one creature type with another. The new creature type can't be Wall. *(This effect lasts indefinitely.)*" This card is vital to the construction, more so than any other.

Also crucial is one of several cards such as **Glamerdye** which read "Change the text of target spell or permanent by replacing all instances of one colour word with another". Similarly, we can change what colour a permanent is with **Prismatic Lace** ("Target permanent becomes the colour or colours of your choice").

For example, the card **Rotlung Reanimator** reads "Whenever Rotlung Reanimator or another Cleric dies, create a 2/2 black Zombie creature token". By casting two copies of **Artificial Evolution** replacing "Cleric" with "Aetherborn" and "Zombie" with "Sliver", and one copy of **Glamerdye** to replace "black" with "white", we can change **Rotlung**

Reanimator to read instead "Whenever Rotlung Reanimator or another *Aetherborn* dies, create a 2/2 *white Sliver* creature token". This allows us to use creature types to track values throughout the computation, killing creature tokens with particular types and using **Rotlung Reanimator** as a conditional logic gate.

It is useful to add extra creature types to some creatures without changing their text box: this can be accomplished with **Olivia Voldaren**, who has the ability "Olivia Voldaren deals 1 damage to another target creature. That creature becomes a Vampire in addition to its other types." We use **Artificial Evolution** to change **Olivia Voldaren** to add the creature type "Assembly-Worker" instead of "Vampire": we will use the type Assembly-Worker to denote infrastructure creatures which will be rendered untargetable by spells.

It should be noted that all these edits only persist for as long as the permanent remains on the battlefield. If an edited permanent changes zone, such as going to the graveyard or the library, these edits are lost. This means that for cards the machine plays from players' hands, we cannot edit them; we need to work with their text as printed.

A.5 Abilities and the Stack

There are many different types of abilities that cards in *Magic: the Gathering* can have. The rules surrounding using abilities get rather complicated, but are crucial to understanding the mechanisms of the constructions in this paper. In this section, we restrict ourselves to explaining the bare minimum required to understand the construction.

Our construction is primarily concerned with *static abilities* and *triggered abilities*. Static abilities are abilities that are "always on" and modify the general rules of the game. For example, **Dread of Night** reads "White creatures get –1/–1." This is a static ability of the **Dread of Night** permanent, affecting all white creatures and reducing their power and toughness by 1 each.

Triggered abilities begin with one of the words "When", "Whenever" or "At". **Rotlung Reanimator** has a triggered ability that reads "Whenever Rotlung Reanimator or another Cleric dies, create a 2/2 black Zombie creature token." **Rotlung Reanimator** is how we will perform many of the functions of the Turing machine. It is a creature with two subtypes: Zombie and Cleric.

Whenever a spell is cast or an ability is activated or triggered, it is first put in a holding area known as the *stack*. When a spell or ability is on the stack, other players may add additional spells or abilities to the stack before the effect *resolves* (takes effect). The stack in *Magic* functions exactly like the data structure of the same name, with the spell or ability put on the stack first being carried out last and the spell or ability put on the stack last being carried out first.

The player whose turn it is is always first to get *priority*, which is permission to add new spells or abilities to the stack, followed by the player whose turn it isn't. Once both players decide to not use their priority to put a spell or ability on the stack, the top effect on the stack is popped and resolves.

Sometimes two triggered abilities will try to go on the stack at the same time. In this case, the order is determined by *Active Player, Nonactive Player (APNAP)* order. The active player is the one whose turn it is. Since this is the order the spells and abilities go on the stack, they will resolve in the reverse order (so the nonactive player's ability resolves first). If both effects are controlled by the same player, that player must choose the order to place them onto the stack. This poses a major limitation throughout the construction, as we aim to eliminate all choices by the players, so we cannot allow a player to control two effects that simultaneously trigger.

Although it is likely possible to simulate a Turing machine using the stack directly, our construction opts to take another tack.

A.6 Phasing

Phasing is an unusual ability some *Magic: the Gathering* cards have that is crucial to our construction. It allows a creature to be treated as if it doesn't exist – in particular, its triggered abilities won't trigger – but it stays on the battlefield, and so edits to its text by **Artificial Evolution** remain. Permanents with phasing toggle between two states each time their controller's turn begins: at the very beginning of a player's turn, all their phased-in permanents with phasing "phase out" (temporarily cease to exist) and all their phased-out permanents "phase in" (come back into existence).

A.7 Counters

There are many effects that can change the power and toughness of creatures. Some of these are temporary and last until the end of turn, while others are permanent. Permanent changes are denoted by *counters* placed on the creatures. In our construction, we will utilize +1/+1 and –1/–1 counters. +1/+1 counters increase the power and toughness of a creature each by 1, while –1/–1 counters decrease them. Pairs of +1/+1 and –1/–1 counters on the same creature cancel out.

A.8 The Structure of a Turn

Play in *Magic: the Gathering* consists of players alternately taking turns. Each turn is divided into phases, with each phase divided into steps such as the *upkeep step*. Many cards in *Magic* say something like "At the beginning of your upkeep..." or "At the beginning of each player's draw step..." At the beginning of each step and phase, the first thing done is always to check for such abilities and put them on the stack. During each of these phases, there is the option to cast spells and activate abilities, but some additionally have game actions players are required to take after all relevant effects have resolved.

The first phase of each turn is the *beginning phase*, which consists of the *untap step*, the *upkeep step*, and the *draw step*. During the untap step players first carry out any phasing effects, and then untap all permanents they control. There are no game actions during the upkeep step, and during the draw step the active player draws one card. Most of the action of the Turing machine happens in the beginning phase.

The second phase is the *first main phase*, where the bulk of the play occurs during a normal game, though nothing happens in the Turing machine. The third phase is the *combat phase*, where combat occurs, but this is not used in our construction. We will also have nothing relevant happen in the fourth phase (the *second main phase*) and only minimal effects during the final *end phase*.

B Tables

Table 1 Game state when the (2, 18) UTM begins.

Card	Controller	Changed text / details
29 Rotlung Reanimator	Bob	See Table 2
7 Xathrid Necromancer	Bob	See Table 2
29 Cloak of Invisibility	Alice	attached to Rotlung Reanimator
7 Cloak of Invisibility	Alice	attached to Xathrid Necromancer
Wheel of Sun and Moon	Alice	attached to Alice
Illusory Gains	Alice	attached to latest tape token
Steely Resolve	Alice	*Assembly-Worker*
2 Dread of Night	Alice	*Black*
Fungus Sliver	Alice	*Incarnation*
Rotlung Reanimator	Alice	*Lhurgoyf, black, Cephalid*
Rotlung Reanimator	Bob	*Lhurgoyf, green, Lhurgoyf*
Shared Triumph	Alice	*Lhurgoyf*
Rotlung Reanimator	Alice	*Rat, black, Cephalid*
Rotlung Reanimator	Bob	*Rat, white, Rat*
Shared Triumph	Alice	*Rat*
Wild Evocation	Bob	None
Recycle	Bob	None
Privileged Position	Bob	None
Vigor	Alice	None
Vigor	Bob	None
Mesmeric Orb	Alice	None
Ancient Tomb	Alice	None
Prismatic Omen	Alice	None
Choke	Alice	None
Blazing Archon	Alice	None
Blazing Archon	Bob	None

Table 2 Text of the Rotlung Reanimators and Xathrid Necromancers encoding the (2, 18) UTM.

Rogozhin's program				Card text					
q_1	1	c_2	Lq_1	Whenever an	Aetherborn	dies, create a		2/2 white	Sliver
q_1	$\overrightarrow{1}$	$\overleftarrow{1}_1$	Rq_1	Whenever a	Basilisk	dies, create a		2/2 green	Elf
q_1	$\overleftarrow{1}$	c_2	Lq_1	Whenever a	Cephalid	dies, create a		2/2 white	Sliver
q_1	$\overrightarrow{1}_1$	1	Rq_1	Whenever a	Demon	dies, create a		2/2 green	Aetherborn
q_1	$\overleftarrow{1}_1$	$\overrightarrow{1}_1$	Lq_1	Whenever an	Elf	dies, create a		2/2 white	Demon
q_1	b	$\overleftarrow{b}$	Rq_1	Whenever a	Faerie	dies, create a		2/2 green	Harpy
q_1	$\overrightarrow{b}$	$\overleftarrow{b}_1$	Rq_1	Whenever a	Giant	dies, create a		2/2 green	Juggernaut
q_1	$\overleftarrow{b}$	b	Lq_1	Whenever a	Harpy	dies, create a		2/2 white	Faerie
q_1	$\overrightarrow{b}_1$	b	Rq_1	Whenever an	Illusion	dies, create a		2/2 green	Faerie
q_1	$\overleftarrow{b}_1$	$\overrightarrow{b}_1$	Lq_1	Whenever a	Juggernaut	dies, create a		2/2 white	Illusion
q_1	b_2	b_3	Lq_2	Whenever a	Kavu	dies, create a	tapped	2/2 white	Leviathan
q_1	b_3	$\overrightarrow{b}_1$	Lq_2	Whenever a	Leviathan	dies, create a	tapped	2/2 white	Illusion
q_1	c	$\overrightarrow{1}$	Lq_2	Whenever a	Myr	dies, create a	tapped	2/2 white	Basilisk
q_1	$\overrightarrow{c}$	$\overleftarrow{c}$	Rq_1	Whenever a	Noggle	dies, create a		2/2 green	Orc
q_1	$\overleftarrow{c}$	$\overrightarrow{c}_1$	Lq_1	Whenever an	Orc	dies, create a		2/2 white	Pegasus
q_1	$\overrightarrow{c}_1$	$\overleftarrow{c}_1$	Rq_2	Whenever a	Pegasus	dies, create a	tapped	2/2 green	Rhino
q_1	$\overleftarrow{c}_1$	$HALT$		Whenever a	Rhino	dies, create a		2/2 blue	Assassin
q_1	c_2	$\overleftarrow{1}$	Rq_1	Whenever a	Sliver	dies, create a		2/2 green	Cephalid
q_2	1	$\overleftarrow{1}$	Rq_2	Whenever an	Aetherborn	dies, create a		2/2 green	Cephalid
q_2	$\overrightarrow{1}$	$\overleftarrow{1}$	Rq_2	Whenever a	Basilisk	dies, create a		2/2 green	Cephalid
q_2	$\overleftarrow{1}$	$\overrightarrow{1}$	Lq_2	Whenever a	Cephalid	dies, create a		2/2 white	Basilisk
q_2	$\overrightarrow{1}_1$	$\overleftarrow{1}_1$	Rq_2	Whenever a	Demon	dies, create a		2/2 green	Elf
q_2	$\overleftarrow{1}_1$	1	Lq_2	Whenever an	Elf	dies, create a		2/2 white	Aetherborn
q_2	b	b_2	Rq_1	Whenever a	Faerie	dies, create a	tapped	2/2 green	Kavu
q_2	$\overrightarrow{b}$	$\overleftarrow{b}$	Rq_2	Whenever a	Giant	dies, create a		2/2 green	Harpy
q_2	$\overleftarrow{b}$	$\overrightarrow{b}$	Lq_2	Whenever a	Harpy	dies, create a		2/2 white	Giant
q_2	$\overrightarrow{b}_1$	$\overleftarrow{b}_1$	Rq_2	Whenever an	Illusion	dies, create a		2/2 green	Juggernaut
q_2	$\overleftarrow{b}_1$	$\overrightarrow{b}$	Lq_2	Whenever a	Juggernaut	dies, create a		2/2 white	Giant
q_2	b_2	b	Rq_1	Whenever a	Kavu	dies, create a	tapped	2/2 green	Faerie
q_2	b_3	$\overleftarrow{b}_1$	Rq_2	Whenever a	Leviathan	dies, create a		2/2 green	Juggernaut
q_2	c	$\overleftarrow{c}$	Rq_2	Whenever a	Myr	dies, create a		2/2 green	Orc
q_2	$\overrightarrow{c}$	$\overleftarrow{c}$	Rq_2	Whenever a	Noggle	dies, create a		2/2 green	Orc
q_2	$\overleftarrow{c}$	$\overrightarrow{c}$	Lq_2	Whenever an	Orc	dies, create a		2/2 white	Noggle
q_2	$\overrightarrow{c}_1$	c_2	Rq_2	Whenever a	Pegasus	dies, create a		2/2 green	Sliver
q_2	$\overleftarrow{c}_1$	c_2	Lq_1	Whenever a	Rhino	dies, create a	tapped	2/2 white	Sliver
q_2	c_2	c	Lq_2	Whenever a	Sliver	dies, create a		2/2 white	Myr

Table 3 60-Card Decklist to play the Turing machine in a Legacy tournament.

Card	Purpose	Card	Purpose
4 Ancient Tomb	Bootstrap	1 Rotlung Reanimator	Logic processing
4 Lotus Petal	Bootstrap	1 Cloak of Invisibility	Logic processing
4 Grim Monolith	Infinite mana device	1 Infest	Logic processing
4 Power Artifact	Infinite mana device	1 Cleansing Beam	Logic processing
4 Gemstone Array	Infinite mana device	1 Soul Snuffers	Logic processing
4 Staff of Domination	Draw rest of deck	1 Illusory Gains	Logic processing
1 Memnarch	Make token copies	1 Privileged Position	Logic processing
1 Stolen Identity	Make token copies	1 Steely Resolve	Logic processing
1 Artificial Evolution	Edit cards	1 Vigor	Logic processing
1 Olivia Voldaren	Edit cards	1 Fungus Sliver	Logic processing
1 Glamerdye	Edit cards	1 Dread of Night	Logic processing
1 Prismatic Lace	Edit cards	1 Wild Evocation	Forced play device
1 Donate	Edit card control	1 Wheel of Sun and Moon	Forced play device
1 Reality Ripple	Edit card phase	1 Shared Triumph	Infinite tape device
1 Djinn Illuminatus	Simplify setup	1 Xathrid Necromancer	Change state
1 Reito Lantern	Simplify setup	1 Mesmeric Orb	Change state
1 Claws of Gix	Simplify setup	1 Coalition Victory	Halting device
1 Riptide Replicator	Set up tape	1 Prismatic Omen	Halting device
1 Capsize	Set up tape	1 Choke	Halting device
1 Karn Liberated	Cleanup after setup	1 Recycle	Remove choices
1 Fathom Feeder	Cleanup after setup	1 Blazing Archon	Remove choices